General Certificate of Education

Mathematics 6360

MFP3 Further Pure 3

Mark Scheme

2009 examination - June series

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation meeting attended by all examiners and is the scheme which was used by them in this examination. The standardisation meeting ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for the standardisation meeting each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed at the meeting and legislated for. If, after this meeting, examiners encounter unusual answers which have not been discussed at the meeting they are required to refer these to the Principal Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of candidates' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available to download from the AQA Website: www.aqa.org.uk

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Key to mark scheme and abbreviations used in marking

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation

√or ft or F	follow through from previous incorrect result	MC	mis-copy
CAO	correct answer only	MR	mis-read
CSO	correct solution only	RA	required accuracy
AWFW	anything which falls within	FW	further work
AWRT	anything which rounds to	ISW	ignore subsequent work
ACF	any correct form	FIW	from incorrect work
AG	answer given	BOD	given benefit of doubt
SC	special case	WR	work replaced by candidate
OE	or equivalent	FB	formulae book
A2,1	2 or 1 (or 0) accuracy marks	NOS	not on scheme
$-x$ EE	deduct x marks for each error	G	graph
NMS	no method shown	c	candidate
PI	possibly implied	sf	significant figure(s)
SCA	substantially correct approach	dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded. However, there are situations in some units where part marks would be appropriate, particularly when similar techniques are involved. Your Principal Examiner will alert you to these and details will be provided on the mark scheme.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

MFP3

Q	Solution	Marks	Total	Comments
1(a)	$y(3.1) = y(3) + 0.1\sqrt{3^2+2+1}$	M1A1		
	$= 2 + 0.1\times\sqrt{12} = 2.3464(10..)$ $= 2.3464$	A1	3	Condone > 4dp if correct
(b)	$y(3.2) = y(3) + 2(0.1)[f(3.1, y(3.1))]$	M1		
	$.... = 2 + 2(0.1)[\sqrt{(3.1^2+2.3464+1)}\,]$	A1F		ft on candidate's answer to **(a)**
	$.... = 2 + 0.2\times3.599499.. = 2.719(89..)$ $= 2.720$	A1	3	CAO Must be 2.720
	Total		**6**	
2	IF is $e^{\int -\tan x\, dx}$	M1		Award even if negative sign missing
	$= e^{\ln(\cos x)}\ (+c)$	A1		OE Condone missing c
	$= (k)\cos x$	A1F		ft earlier sign error
	$\cos x\dfrac{dy}{dx} - y\tan x\cos x = 2\sin x\cos x$			
	$\dfrac{d}{dx}(y\cos x) = 2\sin x\cos x$	M1		LHS as $\dfrac{d}{dx}(y\times \text{IF})$ PI
	$y\cos x = \int 2\sin x\cos x\, dx\, dx$	A1F		ft on c's IF provided no exp or logs
	$y\cos x = \int \sin 2x\, dx$	m1		Double angle or substitution OE for integrating $2\sin x\cos x$
	$y\cos x = -\dfrac{1}{2}\cos 2x(+c)$	A1		ACF
	$2 = -\dfrac{1}{2} + c$	m1		Boundary condition used to find c
	$c = \dfrac{5}{2}$			
	$y\cos x = -\dfrac{1}{2}\cos 2x + \dfrac{5}{2}$	A1	9	ACF eg $y\cos x - 2 + \sin^2 x$ Apply ISW after ACF
	Total		**9**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
3(a)	Centre of circle is $M(3, 4)$	B1		PI
	$A(6, 8)$	B1	2	
(b)(i)	$k = OA = 10$	B1		
	$\tan\alpha = \frac{y_A}{x_A} = \frac{4}{3}$	B1	2	SC " $r = 10$ and $\tan\theta = \frac{8}{6}$ " = B1 only
(b)(ii)	$x^2 + y^2 - 6x - 8y + 25 = 25$	B1		If polar form before expansion award the B1 for correct expansions of both $(r\cos\theta - m)^2$ and $(r\sin\theta - n)^2$ where $(m,n) = (3,4)$ or $(m,n) = (4,3)$
	$r^2 - 6r\cos\theta - 8r\sin\theta = 0$	M1M1		1st M1 for use of any one of $x^2 + y^2 = r^2,\ x = r\cos\theta,\ y = r\sin\theta$ 2nd M1 for use of these to convert the form $x^2 + y^2 + ax + by = 0$ correctly to the form $r^2 + ar\cos\theta + br\sin\theta = 0$
	$\{r = 0$, origin$\}$ Circle: $r = 6\cos\theta + 8\sin\theta$	A1	4	NMS Mark as 4 or 0
	ALTn Circle has eqn $r = OA\cos(\alpha - \theta)$ $r = OA\cos\alpha\cos\theta + OA\sin\alpha\sin\theta$ Circle: $r = 6\cos\theta + 8\sin\theta$	 (M2) (m1) (A1)		 OE
	Total		**8**	

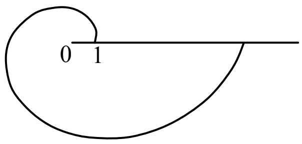
MFP3 (cont)

Q	Solution	Marks	Total	Comments
4	$\int\left(\frac{1}{x}-\frac{4}{4x+1}\right)\mathrm{d}x=\ln x-\ln(4x+1)\{+c\}$	B1		OE
	$\mathrm{I}=\lim_{a\to\infty}\int_1^a\left(\frac{1}{x}-\frac{4}{4x+1}\right)\mathrm{d}x$	M1		∞ replaced by a (OE) and $\lim_{a\to\infty}$
	$=\lim_{a\to\infty}\left[\ln x-\ln(4x+1)\right]_1^a$			
	$=\lim_{a\to\infty}\left[\ln\left(\frac{a}{4a+1}\right)-\ln\frac{1}{5}\right]$	m1		$\ln a-\ln(4a+1)=\ln\left(\frac{a}{4a+1}\right)$ **and** previous M1 scored
	$=\lim_{a\to\infty}\left[\ln\left(\frac{1}{4+\frac{1}{a}}\right)-\ln\frac{1}{5}\right]$	m1		$\ln\left(\frac{a}{4a+1}\right)=\ln\left(\frac{1}{4+\frac{1}{a}}\right)$ **and** previous M1m1 scored
	$=\ln\frac{1}{4}-\ln\frac{1}{5}=\ln\frac{5}{4}$	A1	5	CSO
	Total		**5**	
5(a)	$-k\sin x+2k\cos x+5k\sin x=8\sin x+4\cos x$	M1 A1		Differentiation and subst. into DE
	$k=2$	A1	3	
(b)	Auxl eqn $m^2+2m+5=0$			
	$m=\frac{-2\pm\sqrt{4-20}}{2}$	M1		Formula or completing sq. PI
	$m=-1\pm2\,\mathrm{i}$	A1		
	CF: $\{y_C\}=\mathrm{e}^{-x}(A\sin 2x+B\cos 2x)$	A1F		ft provided m is not real
	GS $\{y\}=\mathrm{e}^{-x}(A\sin 2x+B\cos 2x)+k\sin x$	B1F		ft on CF + PI; must have 2 arb consts
	When $x=0$, $y=1\Rightarrow B=1$	B1F		
	$\frac{\mathrm{d}y}{\mathrm{d}x}=-\mathrm{e}^{-x}(A\sin 2x+B\cos 2x)$ $+\mathrm{e}^{-x}(2A\cos 2x-2B\sin 2x)+k\cos x$	M1		Product rule
	When $x=0$, $\frac{\mathrm{d}y}{\mathrm{d}x}=4\Rightarrow 4=-B+2A+k$	A1		PI
	$\Rightarrow A=\frac{3}{2}$			
	$y=\mathrm{e}^{-x}\left(\frac{3}{2}\sin 2x+\cos 2x\right)+2\sin x$	A1	8	CSO
	Total		**11**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
6(a)(i)	$\mathrm{f}(x)=(9+\tan x)^{\frac{1}{2}}$			
	so $\mathrm{f}'(x)=\frac{1}{2}(9+\tan x)^{-\frac{1}{2}}\sec^2 x$	M1 A1		Chain rule
	$\mathrm{f}''(x)=-\frac{1}{4}(9+\tan x)^{-\frac{3}{2}}\sec^4 x$	M1		Product rule, OE
	$+\frac{1}{2}(9+\tan x)^{-\frac{1}{2}}(2\sec^2 x\tan x)$	A1	4	ACF
(a)(ii)	$\mathrm{f}(0)=3$	B1		
	$\mathrm{f}'(0)=\frac{1}{2}(9)^{-\frac{1}{2}}=\frac{1}{6}$; $\mathrm{f}''(0)=-\frac{1}{4}(9)^{-\frac{3}{2}}=-\frac{1}{108}$	M1		Both attempted and at least one correct ft on c's $\mathrm{f}'(x)$ and $\mathrm{f}''(x)$
	$\mathrm{f}(x)\approx \mathrm{f}(0)+x\,\mathrm{f}'(0)+\frac{1}{2}x^2\,\mathrm{f}''(0)$			
	$(9+\tan x)^{\frac{1}{2}}\approx 3+\frac{x}{6}-\frac{x^2}{216}$	A1	3	CSO AG
(b)	$\frac{\mathrm{f}(x)-3}{\sin 3x}\approx\frac{\frac{x}{6}-\frac{x^2}{216}\ldots}{3x-\frac{(3x)^3}{3!}\ldots}$	M1		Using series expns.
	$\approx\frac{\frac{1}{6}-\frac{x}{216}\ldots}{3-\ldots}$	m1		Dividing numerator and denominator by x to get constant term in each
	$\lim_{x\to 0}\left[\frac{\mathrm{f}(x)-3}{\sin 3x}\right]=\frac{1}{18}$	A1	3	
	Total		**10**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
7(a)	Area $= \frac{1}{2}\int \left(1+6e^{-\frac{\theta}{\pi}}\right)^2 \, d\theta$	M1		Use of $\frac{1}{2}\int r^2 \, d\theta$
	$= \frac{1}{2}\int_0^{2\pi} \left(1+12e^{-\frac{\theta}{\pi}}+36e^{-\frac{2\theta}{\pi}}\right) d\theta$	B1		Correct expansion of $\left(1+6e^{-\frac{\theta}{\pi}}\right)^2$
		B1		Correct limits
	$= \frac{1}{2}\left[\theta - 12\pi e^{-\frac{\theta}{\pi}} - 18\pi e^{-\frac{2\theta}{\pi}}\right]_0^{2\pi}$	m1		Correct integration of at least two of the three terms $1,\ p\,e^{-\frac{\theta}{\pi}},\ q\,e^{-\frac{2\theta}{\pi}}$
	$= \pi\,(16 - 6e^{-2} - 9e^{-4})$	A1	5	ACF
(b)		B1		Going the correct way round the pole
		B1		Increasing in distance from the pole
	End-points $(1, 0)$ and $(e^2, 2\pi)$	B2,1,0	4	Correct end-points B1 for each pair or for 1 and e^2 shown on graph in correct positions
(c)	$e^{\frac{\theta}{\pi}} = 1 + 6\,e^{-\frac{\theta}{\pi}}$	M1		Elimination of r or θ $[r = 1 + \frac{6}{r}]$
	$\left(e^{\frac{\theta}{\pi}}\right)^2 - e^{\frac{\theta}{\pi}} - 6 = 0$	m1		Forming quadratic in $e^{\frac{\theta}{\pi}}$ or in $e^{-\frac{\theta}{\pi}}$ or in r. $[r^2 - r - 6 = 0]$
	$\left(e^{\frac{\theta}{\pi}} - 3\right)\left(e^{\frac{\theta}{\pi}} + 2\right) = 0$	m1		OE
	$e^{\frac{\theta}{\pi}} > 0$ so $e^{\frac{\theta}{\pi}} = 3$	E1		Rejection of negative 'solution' PI $[r = 3]$
	Polar coordinates of P are $(3, \pi \ln 3)$	A1	5	
	Total		**14**	

MFP3 (cont)

Q	Solution	Marks	Total	Comments
8(a)(i)	$\frac{dx}{dt} = 2t$	B1		PI or for $\frac{dt}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$
	$\frac{dx}{dt}\frac{dy}{dx} = \frac{dy}{dt}$	M1		OE Chain rule $\frac{dy}{dx} = \ldots$ or $\frac{dy}{dt} = \ldots$
	$2t\frac{dy}{dx} = \frac{dy}{dt}$ so $2\sqrt{x}\frac{dy}{dx} = \frac{dy}{dt}$	A1	3	AG
(a)(ii)	$\frac{d}{dx}\left(2\sqrt{x}\frac{dy}{dx}\right) = \frac{d}{dx}\left(\frac{dy}{dt}\right) = \frac{dt}{dx}\frac{d}{dt}\left(\frac{dy}{dt}\right)$	M1		$\frac{d}{dx}(f(t)) = \frac{dt}{dx}\frac{d}{dt}(f(t))$ OE eg $\frac{d}{dt}(g(x)) = \frac{dx}{dt}\frac{d}{dx}(g(x))$
	$2\sqrt{x}\frac{d^2y}{dx^2} + x^{-\frac{1}{2}}\frac{dy}{dx} = \frac{1}{2t}\frac{d^2y}{dt^2}$	M1		Product rule OE
	$4t\sqrt{x}\frac{d^2y}{dx^2} + 2tx^{-\frac{1}{2}}\frac{dy}{dx} = \frac{d^2y}{dt^2}$			
	$\Rightarrow 4x\frac{d^2y}{dx^2} + 2\frac{dy}{dx} = \frac{d^2y}{dt^2}$	A1	3	AG Completion
(b)	$4x\frac{d^2y}{dx^2} + 2(1+2\sqrt{x})\frac{dy}{dx} - 3y = 0$			
	$(4x\frac{d^2y}{dx^2} + 2\frac{dy}{dx}) + 2(2\sqrt{x}\frac{dy}{dx}) - 3y = 0$			
	$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = 0$	M1 A1	2	Use of either (a)(i) or (a)(ii) AG Completion
(c)	$\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = 0$ (*)			
	Auxl. Eqn. $m^2 + 2m - 3 = 0$			
	$(m+3)(m-1) = 0$	M1		PI
	$m = -3$ and 1	A1		PI
	GS of (*) $\{y\} = Ae^{-3t} + Be^{t}$	M1		$Ae^{-3x} + Be^{x}$ scores M0 here
	$\Rightarrow y = Ae^{-3\sqrt{x}} + Be^{\sqrt{x}}$	A1	4	
	Total		**12**	
	TOTAL		**75**	